

## ALMOST CONTACT METRIC STRUCTURE ON AN ODD DIMENSIONAL SPHERE

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## ABSTRACT

Parallel fields of planes were used by Patterson (1953) to characterize Kaehler manifolds, and Sasaki (1960) predicted differentiable manifolds having specific structures that are intimately associated with almost contact structures. Additionally, the conformal symmetric tensor of Kaehlerian manifolds was discussed by Negi and Sulochana (2021). We obtained an almost contact metric structure on an odd dimensional sphere in this article. Again, we discover the submanifold of the virtually contact metric manifold in the metric compound structure and the odd-dimensional sphere.

**Keywords:** Riemannian manifold, almost contact manifold, Sasakian manifold, almost complex manifold, Nijenhuis tensors.

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## I. INTRODUCTION

Let  $(\mathcal{M}, F)$  is an  $n$ -dimensional almost contact metric manifold where  $F$  is the nearly complex structure and  $\mathcal{M}$  is an almost contact metric structure. The components of  $K$  and  $F$  with regards to a local coordinate system  $(x^a)$  were now represented by the notation  $\mathcal{M}_{\lambda\mu}$  and  $F_{\lambda}^a$ . If the identity tensor is indicated by  $I = (\delta_{\lambda}^a)$ , then the structure satisfies the equations

$$F^2 = -I; F_{\mu}^{\lambda} F_{\lambda}^a = -\delta_{\mu}^a, \quad (1.1)$$

As well as

$$FGF = G; \tilde{F}_{\nu}^{\lambda} F_{\mu}^a G_{\lambda a} = G_{\nu\mu}. \quad (1.2)$$

If we put the covariant components of  $\tilde{F}$  as

$$F_{\lambda\mu} = GF_{\lambda\mu} = F_{\mu}^a G_{\lambda a}, \quad (1.3)$$

Then  $\lambda$  and  $\mu$  are skew-symmetric in  $F_{\lambda\mu}$ . If we suppose that an  $m$ -dimensional Riemannian manifold  $M$  is immersed isometrically in  $\mathcal{M}$ , we can use the parametric equations

$$x^a = x(y^h), \quad (1.4)$$

Now, using a local coordinate system  $(y^h)$  of  $M$ .

Assume this:

$$B_i^a = \partial_i x^a, \quad (1.5)$$

The  $n$  vectors  $B_i^a$  and  $C_q^a$  span the tangent space  $(M)$  of  $K$  at every point of  $M$ , where  $C_q^a$  is mutually orthogonal unit normal vector fields of  $M$ . Thus,

$$B = (B_B^a) = (B_i^a, C_q^a),$$

Therefore, the metric tensor  $g$  of  $M$  is associated with  $G$  of  $K$ , so

$$g_{ji} = G_{\mu\lambda} B_j^{\mu} B_i^{\lambda}. \quad (1.6)$$

Taking the contravariant components of  $g$  by  $g^{ih}$ , then

$$B_{\lambda}^h = g^{ih} G_{\lambda a} B_i^a, \\ C_{q\lambda} = G_{\lambda a} C_q^a.$$

Consequently, the inverse matrix  $B^{-1}$  of  $B$  is expressed as follows:

$$B^{-1} = (B_{A\lambda}) = \begin{pmatrix} B^h_{\lambda} \\ C_{p\lambda} \end{pmatrix}.$$

Using  $F = B^{-1}FB$ ; we now write

$$(F_B^A) = (B_B^\lambda F_\lambda^a B^A_a) = f_i^h - v_q^h (v_{pi} f_{qp}). \quad (1.7)$$

Next, the constituents of the four types of  $F$  are provided by

$$\begin{aligned} f_i^h &= B_i^\lambda F_\lambda^a B^h_a, \quad v_a^h = -C_a^\lambda F_\lambda^a B^h_a, \\ v_{pi} &= B_i^\lambda F_\lambda^a C_{pa}, \quad f_{qp} = C_q^\lambda F_\lambda^a C_{pa} \end{aligned}$$

Assuming that  $\tilde{F}^* = (F_{\mu\lambda})$  is skew-symmetric,

$$v_{pi} = v_p^h g_{ih}, \quad (1.8)$$

Observe that

$$f_{ji} = B_j^\lambda B_i^a F_{\lambda a}. \quad (1.9)$$

Is skew-symmetric in  $i$  and  $j$ , and

$$f_{qp} = C_q^\lambda C_p^a F_{\lambda a} \quad (1.10)$$

Where  $p$  and  $q$  are skew-symmetric. Thus, a  $(1,1)$ -tensor,  $m$  vector fields, and  $(l-1)/2$  scalar fields on  $M$  are composed of the sets  $f = (f_i^h)$ ,  $v = (v_q^h)$  and  $f^\perp = (f_{qp})$ .

The normal vectors  $C_q^a$  and the tangent vector transforms  $B_i^a$  in  $M$  by  $F$  are now shown as follows:

$$F^a B_i^\lambda = f_i^h B_h^a + v_{pi}^a C_p^a \quad (1.11)$$

and

$$\begin{aligned} F_\lambda^a C_a^\lambda &= -v_a^h B_h^a + \\ f_{qp} C_p^a, \end{aligned} \quad (1.12)$$

On their own range  $m+1, m+2, \dots, n$ , we use the sequel summation technique to repeated lower indices  $p, q, r, \dots$ . Then, the matrix (1.7) satisfies the equation,

$F^2 = -I$ , Meanwhile, the quantities  $f, v$  and  $f^\perp$  are in the relation

$$f_j^i f_i^h = -\delta_j^h + v_{qj} v_q^h, \quad (1.13)$$

$$f_j^i v_{pi} = -v_{qj} f_{qp} = f_{pq} v_{qj}, \quad (1.14)$$

$$v_r^i f_i^h = -f_{ra} v_a^h, \quad (1.15)$$

$$f_{rp} f_{qp} = -\delta_{rp} + v_r^i v_{pi}. \quad (1.16)$$

The relation (1.6) is equivalent to,

$$f_i^a f_i^h g_{ah} + v_{qi} v_{qi} = g_{ii}. \quad (1.17)$$

Let an  $m$ -dimensional Riemannian manifold  $M$  admitting a metric tensor  $g$  and removing the almost contact metric manifold  $\mathcal{M}$  than,  $(1,1)$ -tensor field  $f, m$  vector fields  $v_q$  and  $l(l-2)/2$  scalar fields

$f_{qp}$  satisfy the relations (1.13), (1.14), (1.15), (1.16) and (1.17). Accordingly, the metric compound structure on  $M$  is  $(f, g, v, f^\perp)$ .

We consider that,

$$F = \begin{pmatrix} f_i^h & -v_a^h \\ v_{pi} & f_{qp} \end{pmatrix} \text{ and } G = \begin{pmatrix} g_{ii} & 0 \\ 0 & \delta \end{pmatrix},$$

Then the set  $(F, K)$  defines an almost contact Metric structure in the  $l$  – dimensional Euclidean space  $R^l$  and the product space  $M \times R^l$  of the manifold  $M$ .

## II. ON ALMOST CONTACT METRIC MANIFOLD IN METRIC COMPOUND STRUCTURE

**Theorem 2.1.** Let  $f$  and  $g$  constitute an almost contact metric structure  $(f, g, \xi, \eta)$  on  $M$  and  $(f, g, v, f^\perp)$  be a metric compound structure on  $M$ , then its necessary and sufficient that  $f^\perp$  and  $g^\perp$  constitute an almost contact metric structure  $(f^\perp, g^\perp, v)$  on  $R^l$  at every point of  $M$ .

**Proof.** Assuming that the metric tensor  $g$ , the tensor field  $f$ , and an approximately almost contact metric structure on  $M$  are composed of a covariant vector field  $\eta = (\eta_i)$  and a contravariant vector field  $\xi = (\xi^h)$ , then

$$f^i f^h_i = -\delta^h_j + \eta_j \xi^h, \quad (2.1)$$

$$f^i_h \xi^i = 0, f^h_i \eta_h = 0, \quad (2.2)$$

$$\xi^i \eta_i = 1 \quad (2.3)$$

and

$$f^a_j f^h_i g_{kh} + \eta_j \eta_i = g_{ji}. \quad (2.4)$$

We know that the dimension  $m$  of  $M$  is odd and the rank of  $f = (f_j^i)$  is equal to  $m - 1$ .

Comparison (1.17) with (2.4), we get

$$v_{qj} v_{qi} = \eta_j \eta_i. \quad (2.5)$$

The Above equation shows that the product of the matrix  $(v_{qi})$  with the transpose is of rank 1 and so that the matrix  $(v_{qi})$  by itself is of rank 1.

$$v_{qi} = v_q \eta_i, \quad (2.6)$$

Where  $v_q$  are proportional factors. Since

$$v_{qi} v_q^i = \eta_i \xi^i = 1, \quad (2.7)$$

The equations (1.15) and (1.16) are reduced to

$$f_{qp} v_p = 0, \quad (2.8)$$

and

$$f_{rq} f_{qp} = -\delta_{rp} + v_r v_p, \quad (2.9)$$

Respectively. For every point of  $M$ , where  $g^\perp = (\delta_{qp})$  and the dimension  $l$  of  $R^l$  is odd. the set  $(f^\perp, g^\perp, v)$  forms an almost contact metric structure on  $R^l$  according to equations (2.7), (2.8) and (2.9).

If, on the other hand, the metric compound structure  $(f, g, v, f^\perp)$  introduces an almost contact metric structure  $(f, g, \xi, \eta)$  on  $M$ , we demonstrate that the almost contact metric structure  $(f^\perp, g^\perp, v)$  on  $R^l$  at every point of  $M$ . Then we get.

**Theorem 2.2.** Let us assume that a metric compound structure  $(f, g, v, f^\perp)$  is almost contact metric structure if and only if the  $l$  vector fields  $v_q$  are all parallel to one another, meaning that the matrix  $(v_q^i)$  is of rank 1.

We have applying this by Theorem 2.1.

**Theorem 2.3.** If and only if  $S_{ji}^h = 0$ , then let  $(f, g, v, f^\perp)$  be an almost contact metric compound structure on  $M$  and let  $(f, g, \xi, \eta)$  be an almost contact metric structure on  $M$ . Thus, Nijenhuis tensor is vanish.

**Proof.** The Nijenhuis tensor of the metric compound structure (1.18) in  $M \times R^l$  is defined by taking  $\partial_q$  as null operators and

Representing  $\partial_j = \partial/\partial y^j$ .

$$S_{CB}^A = F_C^E (\partial_E F^A - \partial_B F_E^A) - F_B^E (\partial_E F_C^A - \partial_C F_E^A).$$

Using (1.18), we can write down  $S_{CB}^A$  as the followings;

$$\begin{aligned} S_{ji}^h &= f_j^l (\partial_l f_i^h - \partial_i f_l^h) - f_i^l (\partial_l f_j^h - \partial_j f_l^h) + v_{js} \partial_i v_s^h - v_{is} \partial_j v_s^h, \\ S_{jip} &= f_j^l (\partial_l v_{pi} - \partial_i v_{pl}) - f_i^l (\partial_l v_{pj} - \partial_j v_{pl}) - v_{js} \partial_i f_{sp} + v_{is} \partial_j f_{sp}, \\ S_{jq}^h &= -f_j^l \partial_l v_q^h + v_q^l (\partial_l f_j^h - \partial_j f_l^h) + f_{qs} \partial_j v_s^h, \\ S_{jqp} &= f_j^l \partial_l f_{qp} + v_q^l (\partial_l v_{pj} - \partial_j v_{pl}) + f_{qs} \partial_j f_{sp}, \\ S_{rq}^h &= v_r^l \partial_l v_q^h - v_q^l \partial_l v_r^h, \\ S_{rqp} &= -v_r^l \partial_l f_{qp} + v_q^l \partial_l f_{rp}. \end{aligned} \quad (2.10)$$

Consequently, the above mentioned expressions are simplified to the metric compound structure  $(f, g, v, f^\perp)$  which yields an almost contact metric structures

$$\begin{aligned} &(f, g, \xi, \eta) \text{ on } M \text{ and } (f^\perp, g^\perp, v) \text{ on } R^l. \\ S_{ji}^h &= f_j^l (\partial_l f_i^h - \partial_i f_l^h) - f_i^l (\partial_l f_j^h - \partial_j f_l^h) + \eta_j \partial_i \xi^h - \eta_i \partial_j \xi^h, \\ S_{jip} &= [f_j^l (\partial_l \eta_i - \partial_i \eta_l) - f_i^l (\partial_l \eta_j - \partial_j \eta_l)] v_p + (f_j^l \eta_i - f_i^l \eta_j) \partial_l v_p + (\eta_j \partial_i v_s - \eta_i \partial_j v_s) f_{sp}, \\ S_{jq}^h &= [\xi^l (\partial_l f_j^h - \partial_j f_l^h) - f_j^l \partial_l \xi^h] v_q - (f_j^l \partial_l v_q + f_{qs} \partial_j v_s) \xi^h, \\ S_{jqp} &= (\xi^l \partial_l \eta_j - \xi^l \partial_j \eta_l) v_q v_p + (\eta_j \xi^l \partial_l v_p - \partial_j v_p) v_q + f_j^l \partial_l f_{qp} + f_{qs} \partial_j f_{sp}, \\ S_{rq}^h &= (v_r \xi^l \partial_l v_q - v_q \xi^l \partial_l v_r) \xi^h, \\ S_{rpq} &= -v_r \xi^l \partial_l f_{qp} + v_q \xi^l \partial_l f_{rp}, \end{aligned} \quad (2.11)$$

Because  $v_q v_q = 1$  and  $v_q \partial_j v_q = 0$ .

Now, the Nijenhuis tensors of the almost contact kaehlerian structure  $(f, g, \xi, \eta)$  are given by ([4])

$$\begin{aligned} N_{ji}^h &= f_j^l (\partial_l f_i^h - \partial_i f_l^h) - f_i^l (\partial_l f_j^h - \partial_j f_l^h) + \eta_j \partial_i \xi^h - \eta_i \partial_j \xi^h, \\ N_{ji} &= f_j^l (\partial_l \eta_i - \partial_i \eta_l) - f_i^l (\partial_l \eta_j - \partial_j \eta_l), \\ N_j^h &= \xi^l (\partial_l f_j^h - \partial_j f_l^h) - f_j^l \partial_l \xi^h, \\ N_j &= \xi^l \partial_l \eta_j - \xi^l \partial_j \eta_l. \end{aligned} \quad (2.12)$$

Comparing (2.11) with (2.12), we have the equations

$$\begin{aligned} N_{ji}^h &= S_{ji}^h, N_{ji} = S_{jip} v_p, \\ N_j^h &= S_{jq}^h v_q, N_j = S_{jqp} v_q v_p. \end{aligned} \quad (2.13)$$

Thus, we drive the following from (3.4).

### III. SUBMANIFOLD OF ALMOST CONTACT METRIC MANIFOLD IN ODD-DIMENSIONAL SPHERE

A pseudo-umbilical submanifold of an even-dimensional Almost Contact metric manifold  $\mathcal{M}^{2n+4}$  that satisfies  $\lambda^2 + \mu^2 + \nu^2 = 1$  with  $(f, g, u, v, w, \lambda, \mu, \nu)$  structure, where a sphere and a complicated cone with a generator intersect.

Consider a  $(2n-1)$ -dimensional Riemannian manifold  $M^{2n-1}$  that is isometrically immersed in  $S^{2n+1}$  by an immersion  $i: M^{2n-1} \rightarrow S^{2n+1}$  and covered by a system of coordinate neighborhoods  $\{V; y^a\}$ .

Assume that

$$B^h_c = \partial x^h_c, \partial_c = \partial/\partial y^c.$$

Let  $D^h$  and  $E^h$  be mutually orthogonal unit normal to  $S^{2n+1}$ , and let each  $B^h_c$  is a  $2n-1$  linearly independent vector of  $S^{2n+1}$  tangent to  $M^{2n-1}$ . let  $g_{cb}$  represents the component of the induces metric tensor of  $M^{2n-1}$ , then  $g_{cb} = g_{ji}B^j_i B^i_c$ .

After the transform of

$$B^j_i, j \text{ and } E^j \text{ by } f^h_i,$$

We have

$$\begin{aligned} f^h_j B^j_c &= f^a_c B^h_a + v_c D^h_c + w_c E^h_c, \\ \{ f^h_j D^j_c &= -v^a B^h_a - \lambda E^h_c, \\ f^h_i E^j &= -w B^j_a + \lambda D^j_c, \end{aligned} \quad (3.1)$$

Where  $f^a_c$  indicates the components of a tensor field of type (1.1) in  $M^{2n-1}$  and  $u_c, v_c$  and  $w_c$  are 1-forms associated with  $u^a, v^a$  and  $w^a$  respectively. The vector field  $u^h$  is entered as follows:

$$u^h = u^a B^h_a + \mu D^h_c + \nu E^h_c, \quad (3.2)$$

Both  $\mu$  and  $\nu$  are functions in  $M^{2n-1}$  since  $u^a$  is a vector field.

Using the operator  $f^k_h$  in (3.1) and (3.2) respectively, then

$$f^e_b f^a_e = -\delta^a_b + u^a_b u^a + v^a_b v^a + w^a_b w^a, \quad (3.3)$$

$$\begin{aligned} f^a_e u^e &= \mu v^a + \nu w^a, \\ \{ f^a_e v^e &= -\mu u^a + \lambda w^a, \\ f^a_e w^e &= -\nu u^a - \lambda v^a, \end{aligned} \quad (3.4)$$

$$\begin{aligned} \text{Or } u^e f^e_c &= -\mu v_c - \nu w_c, & v^e f^e_c &= \mu u_c - \lambda w_c, \\ \lambda w^e_c, w^e f^e_c &= \nu u_c + \lambda v_c, \\ u^e u^e &= 1 - \mu^2 - \nu^2, & u^e v^e &= -\lambda v, \\ \{ u^e w^e &= \lambda \mu, & v^e v^e &= 1 - \lambda^2 - \mu^2, \\ v^e w^e &= -\mu \nu, & w^e w^e &= 1 - \lambda^2 - \nu^2, \end{aligned} \quad (3.5)$$

Since  $f^a_{cb} = f^a_g$  is skew-symmetric.

$$f^e f^d g_{cb} = g_{ed} - u^u_{cb} - v^v_{cb} - w^w_{cb}.$$

The equation of Gauss for  $M^{2n-1}$  can be written as follows, representing the operator  $\nabla_C$  for the vander Waerden- Bortotti covariant differentiation:

$$\nabla_c B_b^h = k_{cb} D^h + l_{cb} E^h, \quad (3.6)$$

Where  $k_{cb}$  and  $l_{cb}$  are the second fundamental tensors with respect to  $D^h$  and  $E^h$ , respectively.

The equation of Weingarten are given by

$$\nabla_c D^h = -k_c^b B_b^h + l_c^h E^h, \quad (3.7)$$

$$\nabla_c E^h = -l_c^b B_b^h - l_c D^h, \quad (3.8)$$

Where  $k_c = k_{ca} g^{ab}, l_c^b = l^{ab} g_{ab}, (g^{ab}) = (g_{ab}), l_c$

being the third fundamental tensor. The normal bundle if  $l_c$  vanishes identically. After that, the Gauss equation is obtained from (1.2) by

$$K_{acb}^a = \delta_a^a g_{cb} - \delta_c^a g_{ab} + k_a^a k_{cb} - k_c^a k_{ba} + l_a^a l_{cb} - l_c^a l_{ba}.$$

Considering (1.5), (2.2), (2.6) (2.7), and (2, 8), and differentiating (2.1) and (2.2) covariantly along  $M^{2n-1}$ , we have

$$\nabla_b f_c^a = -g_{bc} u^a + \delta_b^a u_c + k_b^a v_c - k_{bc} w^a, \quad (3.9)$$

$$\nabla_b v_c = -\mu g_{bc} + \lambda l_{bc} - k_{ba} f_c^a + l_b w_c, \quad (3.10)$$

$$\nabla_b w_c = -v g_{bc} - \lambda k_{bc} - l_{ba} f_c^a - l_b v_c, \quad (3.11)$$

$$\begin{aligned} \nabla_b u^a &= f_b^a + \mu k_b^a + v l_b^a, \\ \nabla_b \lambda &= k_b^a w_a - l_{ba} v^a, \\ \nabla_b \mu &= v_b - k_{ba} u^a + v l_b, \\ \nabla_b v &= w_b - l_{ba} u^a - \mu l_b. \end{aligned} \quad (3.12)$$

**Theorem3.1.** An even-dimensional Almost contact metric manifold  $\mathcal{M}^{2n+4}$  with a pseudo-umbilical submanifold  $M^{2n+1}$  whose  $(f, g, u, v, w, \lambda, \mu, \nu)$ - structure satisfies  $\lambda^2 + \mu^2 + \nu^2 = 1$ . Then, a complex cone with a generator as a normal vector intersects a sphere at  $M^{2n+1}$ .

**Proof.** Consider that an almost contact metric structure  $(f_b^a, g_{cb}, u^a)$  is admitted by  $M^{2n-1}$ , which is defined by

$$f_b^a f_c^a = -\delta_b^a + u_b u^a, \quad (3.13)$$

$$u_a u^a = 1. \quad (3.14)$$

Using (3.3), (3.5), and (3.13), we get

$$(1 - \lambda - \mu^2) + (1 - \lambda^2 - \nu^2) = 0. \quad (3.15)$$

Equations (3.5) and (3.14) indicates that

$$\mu = 0, \nu = 0. \quad (3.16)$$

Putting (3.16) into (3.15), then

$$\lambda^2 = 1. \quad (3.17)$$

By (3.5), Adding equation (3.16) and (3.17), then

$$v^a = 0, w^a = 0. \quad (3.18)$$

Equations (3.9) and (3.12) implies that

$$f_b^a f_c^a = -\delta_b^a + u_b u^a, \quad u_a u^a = 1.$$

Thus, a sasakian structure is expressed by the structure

$$(f^a_c, g_{cb}, u^a)$$

Hence, we get:

**Theorem 3.2.** An odd – dimensional submanifold  $M^{2n-1}$  of a sphere  $S^{2n-1}$  of codimension 2. As a submanifold of codimension 2 of an Almost contact metric manifold  $\mathcal{M}^{2n+2}$  is pseudo-umbilical. If  $2n - 1$  is minimal.

**Proof.** By substituting (3.16) and (3.18), we obtain

$$\lambda l_{bc} - k_{ba} f^a_c = 0,$$

Now, using (3.17), and contracting the previous equation with respect to b and c,

$$l^b_b = 0, \quad (3.19)$$

Since,  $k_{cb}$  are symmetric and  $f_{cb}$  is skew- symmetric with respect to b and c. Using (3.11), (3.16), (3.18) and (3.17),

$$k^b_b = 0. \quad (3.20)$$

Therefore, the mean curvature vector is certain by

$$H^a = \frac{1}{2n-1} g^{ab} \nabla_b B^a_a = \frac{1}{2n-1} (k^a_a u^a + l^a_a u^a).$$

Using (3.19) in (3.20), then we have  $H^h = 0$ , The above expression indicates that  $M^{2n-1}$  is a minimal submanifold.

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