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RĀŚIKAṂ SŪTRAS IN VYAVAHĀRA GAṆITA: CLASSICAL INDIAN METHODS FOR SOLVING FINANCIAL AND DAILY-LIFE PROBLEMS FROM PROPORTION RULES TO BARTER EXCHANGE

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ABSTRACT
Classical Indian mathematics exhibits a strong tradition of applied problem solving through concise and systematic sutras, particularly within the framework of Rāśikaṃ*. These sutras played a crucial role in addressing practical problems related to finance and everyday life, forming an integral part of* Vyavahāra Gaṇita*. This paper explores the sutras associated with* Rāśikaṃ *as presented in classical-era mathematical texts and examines their application in solving problems involving trade, interest, profit and loss, proportional distribution, wages, and routine daily household calculations.*

Through an analytical study of selected sūtras based methods, the paper highlights how principles such as proportionate reasoning, arithmetic operations, and ratio founded computation were efficiently employed to meet real world socio economic needs in Ancient Classical Indian society. The study demonstrates that these sutras not only enabled quick and accurate calculations but also reflected a deep integration of mathematics with daily routine human activities. By revisiting Rāśikaṃ *based problem solving techniques, the paper features their historical significance and pedagogical applicability, suggesting their potential value in enhancing conceptual understanding and contextual learning in present day mathematics education.*

Keyword: *Rāśikaṃ; Vyavahāra Gaṇita; Āryabhaṭīya; Brāhmasphuṭa siddhānta; Pramāna; Icchā*

1. INTRODUCTION
The tradition of ***Vyavahāra Gaṇita*** in ancient classical Indian mathematics represents a rich amount of applied arithmetic devoted to business, commerce, taxation, exchange, measurement and everyday transactions. Unlike purely theoretical works, these *sūtras* reveal how mathematical thinking functioned within social and economic life. Significant to this tradition are the ***Rāśikaṃ*** methods procedures based on proportional reasoning that enabled merchants, administrators, and scholars to solve practical problems containing price, quantity, interest, and barter.

Ancient treatises such as the Āryabhaṭīya, the Brāhmasphuṭa siddhānta, and later works like the Līlāvatī present systematic formulations of proportional rules including the *Trairāśika* (rule of three), inverse proportion, compound proportion (rules of five, seven, nine, and eleven), and barter exchange techniques. These methods demonstrate a sophisticated understanding of ratios, scaling, and transactional equivalence long before their formal algebraic expression in modern notation.

This paper examines selected *Rāśikaṃ* associated *sūtras* as computational tools within *Vyavahāra* Gaṇita, highlighting how they structured financial, logical reasoning and problem solving in the ancient classical Indian period. By putting these procedures within their textual and practical contexts, the study seeks to illuminate the continuity between mathematical theory and existed economic practice in modern India.

2. *VYAVAHĀRA* GAṆITA AS APPLIED ARITHMETIC TRADITION
The branch of ***Vyavahāra Gaṇita*** represents the practical dimension of ancient classical Indian mathematics, focusing on trade, finance, taxation, wages, measurement, and exchange. Unlike abstract astronomy or algebra, this tradition addressed the needs of merchants, administrators, and craftsmen. Mathematical operations were framed as rule based procedures (*sūtras*), enabling quick calculation without symbolic algebra. Treatise such as the Āryabhaṭīya and the Brāhmasphuṭa siddhānta exhibit that proportional reasoning was already systematized and widely applied.

3. *RĀŚIKAṂ* AND PROPORTIONAL REASONING
The term ***Rāśikaṃ*** denotes methods based on quantities arranged in proportional sets. The most fundamental of these is the ***Trairāśika*** (Rule of Three), where the relation between *pramāṇa* (given quantity), *phala* (result), and *icchā* (desired quantity) determines the unknown. This method allowed calculation of prices, wages, and quantity requirements through basic multiplication and division.

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According to modern notation if $a \rightarrow b$ *then* $c \rightarrow x$ then $x = \frac{c \times b}{a}$, in ancient this was represented by a sutra II-26-27 of Āryabhatīya and stated below:

Sutra: 1. Āryabhatīya: II-26-27.

त्रैराशिकफलराशिं तमथेच्छाराशिनाहतं कृत्वा ।
लब्धं प्रमाणभजितं तस्मादिच्छाफलमिदं स्यात् ॥
छेदाः परस्परं हता भवन्ति गुणकार भागहाराणां ।
छेदगुणं सच्छेदं परस्परं तत्सवर्णत्वम् ॥ *Aryabhatiya II-26-27*

Translation:
Phala is first multiplied by iccha and then divided by pramana. The quotient which one get is the phala for resultant iccha.

Brahmagupta method was moreover similar to Āryabhata and was stated in Sutra XII-10.

Sutra: 2. Brahmasphuta: XII-10.

त्रैराशिके प्रमाणं फलमिच्छाद्यन्तयोः सदृशराशी ।
इच्छाफलेन गुणिता प्रमाणभक्ता फलं भवति ॥ *Brahmasphuta Siddhanta XII-10*

Translation:
Pramana i.e. argument, phala i.e. fruit and iccha i.e. requisition are the three elementary terms of Rule of three. The fruit of iccha is phala multiplied by iccha and divided by pramana.

Other mathematicians, including **Bhāskara II**, **Nārāyaṇa Paṇḍita**, and **Āryabhaṭa II**, adopted similar procedures for proportional computation. Notably, Āryabhaṭa II employed the terminology *māna*, *vinimaya*, and *icchā* in place of the earlier terms *pramāṇa*, *phala*, and *icchā*. To explain the application of this method, let us consider a problem from *Pāṭīgaṇita* that validates the working of the Rule of Three

Problem:
How much will 9 palas and 1 karsa of Sandal wood cost when the 1 pala and 1 karsa of sandal wood can be obtained for ten and half panas?

Solution:
Let p = pramana (argument)=1 pala 1 karsa = $1\frac{1}{4} = \frac{5}{4}$ *palas (as 4 karsa = 1 pala)*

f = phala (fruit) = $10\frac{1}{2} = \frac{21}{2}$ *palas*

I = iccha (Requesition) = 9 palas 1 karsa = $9\frac{1}{4} = \frac{37}{4}$ *palas*

Applying Rule of three

In the form of mixed fraction

1	10	9
1	1	1
4	2	4

After converting into proper fraction

5	21	37
4	2	4

Now applying Rule of three phala (result) $=\frac{f\times i}{p}$

21	5
2	4
37	
4	

$$=\frac{{}^{21}/_{2}\times{}^{37}/_{4}}{{}^{5}/_{4}}$$

After transferring denominator

21	5
4	2
37	4

$$=\frac{21\times4\times37}{5\times2\times4}=\frac{3108}{40}=\frac{777}{10}\ palas$$

Brahmagupta extended this idea to the **Vyasta Trairāśika** (inverse rule of three), applicable to problems involving inverse variation such as distribution or measurement changes. These rules illustrate a theoretical grasp of direct and inverse proportionality centuries before formal algebraic notation.

Sutra: 3. Brahmasphuta: XII-11.

व्यस्त त्रैराशिक फलमिच्छा भक्तः प्रमाण फलघातः ।
त्रैराशिकादिषु फलं विषमेष्वेकादशान्तेषु ॥ *Brahmasphuta Siddhanta XII-11*

Translation:
First divide phala (fruit) with iccha (requisition) and then multiply it with pramana (argument) this gives Vyasta trairasika i.e. Inverse rule of three.

To illustrate the application of the above *sūtra*, let us consider an example from the *Līlāvatī* of **Bhāskara II**, where a similar proportional problem is presented.

Sutra: 4. Lilavati-Terashik: 3.

सप्ताढकेन मानेन राशौ सस्यस्य मापिते ।
यदि मानशतं जातं तदा पञ्चाढकेन किम् ? ॥ ३ ॥ *Lilavati-Terashik-sutra3*

Explanation:
With a measure of 7 adhakas (*pramana*) a certain quantity of grain measures 100 units (*phala*) find how many units will be there if the measure is 5 adhakas (*icha*)?

pramana	*Iccha*	
7	5	
100	x	← *phala*

According to Sutra XII-11 of Brahmagupta the answer is equal to *phala* divided by *icha* and then multiplied with *pramana* $=\frac{100}{5}\times7=140\ units.$

4. Compound Proportion: Rules of Five to Eleven
The proportional framework expanded into compound structures known as ***Pañca rāśika*** (Rule of five), ***Sapta rāśika*** (Rule of seven), ***Nava rāśika*** (Rule of nine), and ***Ekādaśa rāśika*** (Rule of eleven), corresponding to rules involving multiple related quantities. These procedures enabled solutions to problems involving interest, transport costs, dimensions of goods, taxations, and production scaling.

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In works like the Līlāvatī, such problems appear in commercial and skills contexts, demonstrating the adaptability of proportional reasoning to multivariable situations. The computational approach multiplying one set of terms and dividing by another efficiently anticipates modern dimensional analysis and scaling techniques.

Prior to examining examples of compound proportion, let us first cite Sūtras XII.11–12 from the *Brāhmasphuṭa siddhānta*, as they provide the foundational rule employed in these calculations.

Sutra: 5. Brahmasphuta: XII-11-12.

व्यस्त त्रैराशिक फलमिच्छा भक्तः प्रमाणफलघातः ।
त्रैराशिकादिषु फलं विषमेष्वेकादशान्तेषु ॥
फलसंक्रमणमुभयतो बहुराशि वधोऽल्पवधहृतो ज्ञेयम् ।
सकलेष्वेवं भिन्नेषूभयतश्छेदसंक्रमणम् ॥ *Brahmasphuta Siddhanta XII 11-12*

Translation:
For odd term starting with three to eleven the result is obtained by swapping the phāla of both sides and then dividing the product of the larger set by the product of smaller set.

Let us consider a few examples from ancient treatises illustrating the method of compound proportion.

Rule of Five:
The procedure for the Rule of Five may be understood from the following sutra:

Sutra: 6. Lilavati Prashvarashyadikam: 2.

सत्र्यंशमासेन शतस्य चेत् स्यात् कलान्तरं पञ्च सपञ्चमांशाः ।
मासैस्त्रिभिः पञ्चलवाधिकैस्तत् सार्धद्विषष्टेः फलमुच्यतां किम् ? ॥ २ ॥ *Lilavati-Prashvarashyadikam-2*

Explanation:
If $5\frac{1}{5}$ is the interest on 100 for $\frac{4}{3}$ months, then find the interest on $62\frac{1}{2}$ for $3\frac{1}{5}$ months.

By using rule of five

100	$62\frac{1}{2}$	
$\frac{4}{3}$	$3\frac{1}{5}$	
$5\frac{1}{5}$	x	← *phala*

Now swap the phala as stated in the sutra *XII-11-12* of *Brahmasphuta Siddhanta.*

100	$62\frac{1}{2}$
$\frac{4}{3}$	$3\frac{1}{5}$
x	$5\frac{1}{5}$

Now multiply both the set pramana as well as iccha paksa. The product of pramana paksa is $\frac{4}{3} \times 100 = \frac{400}{3}$ *and the product of iccha paksa is* $\frac{125}{2} \times \frac{16}{5} \times \frac{26}{5} = \frac{52000}{50} = 1040$

According to sutra divide the larger number of products term by smaller number of products term to get phala i.e.

$$phala = \frac{1040}{400/3} = \frac{1040 \times 3}{400} = \frac{39}{5} = 7\frac{4}{5}$$

∴ *Iccha phala is* $7\frac{4}{5}$.

 ISSN 2394 - 7780

Rule of Seven (*Saptarashi*):
The *sūtra* stated below forms the basis for understanding the Rule of Seven.

Sutra: 7. Lilavati-Prashvarashyadikam: 3.

विस्तारे त्रिकराः कराष्टकमिता दैर्घ्ये विचित्राश्च चे-
द्रूपैरुत्कटपट्टसूत्रपटिका अष्टौ लभन्ते शतम् ।
दैर्घ्ये सार्धकरत्रयाऽपरपटी हस्तार्धविस्तारिणी
तादृक् किं लभते ? द्रुतं वद वणिक् ! वाणिज्यकं वेत्सि चेत् ॥ *Lilavati-Prashvarashyadikam-3*

Explanation: 8 pieces of multi coloured embroidered cloth each measuring 3 cubits × 8 cubits are available for 100 niskas , O! business man if you are very good in trade then tell me fast the price of a piece $of\ 3\frac{1}{2}\ cubits \times \frac{1}{2}\ cubit.$

By using rule of seven

Pramana	*Iccha*	
8	*1*	
3	$3\frac{1}{2}$	
8	$\frac{1}{2}$	
100	*X*	← *phala*

Now swap the phala as stated in the sutra *XII-11-12* of *Brahmasphuta Siddhanta.*

Pramana	*Iccha*	
8	*1*	
3	$3\frac{1}{2}$	
8	$\frac{1}{2}$	
X	*100*	← *phala*

Now multiply both the set pramana as well as iccha paksa. The product of pramana paksa is 8 $\times 3 \times 8 = 192$ *and the product of iccha paksa is 1* $\times \frac{7}{2} \times \frac{1}{2} \times 100 = 175$

According to sutra divide the larger number of products term by smaller number of products term to get phala i.e.

$$phala = \frac{175}{192}\ niskas$$

∴ *Iccha phala is* $\frac{175}{192}\ niskas.$

Rule of Nine (*Navrashi*):
The method of the Rule of Nine is explained in the *sūtra* given below.

Sutra: 8. Lilavati-Prashvarashyadikam: 4.

पिण्डे येऽर्कमिताङ्गुलाः किल चतुर्वर्गाङ्गुला विस्तृतौ
पट्टा दीर्घतया चतुर्दशकरास्त्रिंशल्लभन्ते शतम् ।
एता विस्तृतिपिण्डदैर्घ्यमितयो येषां चतुर्वर्जिताः
पट्टास्ते वद मे चतुर्दश सखे! मूल्यं लभन्ते कियत् ? ॥ १ ॥ *Lilavati-Prashvarashyadikam-4*

Explanation:
O! Friend, 30 planks of wood whose width is 12 fingers, breadth is 16 fingers and length is 14 cubits costs 100 niskas then find the cost of 14 planks of wood whose width is 8 fingers, breadth is 12 fingers and length is 10 cubits i.e. each 4 measure less than the former?

By using rule of nine.

Pramana	*Iccha*	
30	14	
12	8	
16	12	
14	10	
100	X	← *phala*

Now swap the *phala* as stated in the sutra *XII-11-12* of *Brahmasphuta Siddhanta.*

Pramana	*Iccha*	
30	14	
12	8	
16	12	
14	10	
X	100	← *phala*

Now multiply both the set, pramana as well as iccha paksa. The product of pramana paksa is $30 \times 12 \times 16 \times 14 = 80640$ *and the product of iccha paksa is* $14 \times 8 \times 12 \times 10 \times 100 = 1344000$

According to sutra divide the larger number of products term by smaller number of products term to get phala i.e.

$$phala = \frac{1344000}{80640} = \frac{50}{3} = 16\frac{2}{3} niskas$$

$\therefore$ *Iccha phala is* $16\frac{2}{3} niskas.$

Rule of Eleven:

The problem to explain rule of eleven is stated in sutra listed below:

Sutra: 9. Lilavati-Prashvarashyadikam: 5.

पट्टा ये प्रथमोदितप्रमितयो गव्यूतिमात्रे स्थिता-
स्तेषामानयनाय चेच्छकटिनां द्रम्माष्टकं भाटकम् ।
अन्ये ये तदनन्तरं निगदिता माने चतुर्वर्जिता-
स्तेषां का भवतीति भाटकमिति गव्यूतिषट्के वद ॥ १ ॥ *Lilavati-Prashvarashyadikam-5*

Explanation:
If the transport charges for carrying the 30 planks of previous example i.e. 14 cubits × 16 fingers × 12 fingers through the distance of 1 *gavyauti* (i.e. 2 *kosa*) is 8 *dramm (1 dramm =16 paise)* then find the charges for

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transporting the second set of 14 planks, which was measuring 4 units less in each dimension i.e. 10 cubits × 14 fingers × 8 fingers through a distance of 6 *gavyauti* (i.e. 12 *kosas*).

By using rule of eleven.

Pramana	*Iccha*	
30	14	
14	10	
16	12	
12	8	
1	6	
8	X	← *phala*

Now swap the phala as stated in the sutra *XII-11-12* of *Brahmasphuta Siddhanta.*

Pramana	*Iccha*	
30	14	
14	10	
16	12	
12	8	
1	6	
x	8	← *phala*

Now multiply both the set pramana as well as iccha paksa. The product of pramana paksa is $30 \times 14 \times 16 \times 12 \times 1 = 80640$ *and the product of iccha paksa is* $14 \times 10 \times 12 \times 8 \times 6 \times 8 = 645120$

According to sutra divide the larger number of products term by smaller number of products term to get phala i.e.

$$phala = \frac{645120}{80640} = 8\ dramm$$

$\therefore$ *Iccha phala is* $8\ dramm.$

5. Barter and Exchange Calculations

Beyond proportional rules, Ancient mathematicians also addressed market exchange through **Bhāṇḍa–Pratibhāṇḍa** (barter methods). These problems considered equivalence of goods with different prices and quantities. By rearranging price and quantity terms and applying proportional logic, mathematicians could determine fair exchange values.

Such examples reveal how mathematical reasoning was embedded in economic transactions, illustrating a direct link between arithmetic procedures and marketplace practice.

Sutra: 10. Brahmasphuta: XII-13.

प्राग्मूल्य व्यस्त्यासो भाण्डप्रतिभाण्डकेऽप्युक्तसमम् ।
परिकर्माण्यष्टानां व्यवहाराणामभिहितानि ॥ १३ ॥ *Brahmasphuta Siddhanta XII 13*

Method states as first swap the prices then swap the *phala* as done in rule five then find the product of both the set the result is the quotient which one get after dividing larger set with smaller set.

Steps involved in Barter & Exchange method is listed below:

Take pramana paksha p_1, a_1, q_1 (phala) and iccha paksa p_2, a_2, x

pramana	*Iccha*	
p_1	p_2	
a_1	a_2	
q_1	0	← *phala*

On the basis of sutra just swap prices first.

pramana	*Iccha*	
p_2	p_1	
a_1	a_2	
q_1	x	← *phala*

Now swap the phala

pramana	*Iccha*	
p_2	p_1	
a_1	a_2	
x	q_1	← *phala*

Multiply both the set

Product of first set is $p_2 a_1 x$

Product of second set is $p_1 a_2 q_1$ $\therefore$ $result\ or\ phala\ x = \frac{p_1 a_2 q_1}{p_2 a_1}$

Example to support Barter and Exchange method:

Problem: *if 2 palas of sandal wood cost 20 panas and 4 palas of turmeric costs 5 panas then how many turmeric stick will be a fair exchange for 4 palas of sandal wood?*

Solution:
Pramana paksa – 2 palas, 20 panas (price), 4 pala (quantity).

Iccha paksa- 4 palas, 5 panas, x pala.

Then according to ancient notation.

20	5	→ *price*
2	4	
4	X	→ *quantity*

First swap the price.

5	20	→ *price*
2	4	
4	x	→ *phala*

Now follow the rule of five i.e. swap the phala.

5	20	→ *price*
2	4	
x	4	→ *phala*

Product of pramana paksa =10

Product of iccha paksa= 320

$$phala = \frac{larger\ product}{smaller\ product} = \frac{320}{10} = 32\ palas$$

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Brahmagupta had put all the logistics or *Parikarma* in systematic way and sutras with proper numbering one below the other. So one can easily track and trace the sutra of particular logistics or *Parikarma*.

Table 1: Rāśikaṃ Rules and Their Practical Application Areas in Vyavahāra-Gaṇita

Rule / Method	Sanskrit Term	Mathematical Nature	Typical Variables	Application Area	Example Contexts
Rule of Three	*Trairāśika*	Direct proportion	Quantity, price, result	Trade, pricing, wages	Cost of goods, grain pricing, salary computation
Inverse Rule of Three	*Vyasta Trairāśika*	Inverse proportion	Measure vs quantity, time vs work	Distribution, measurement changes	Grain measured by vessels, workforce scaling
Rule of Five	*Pañca Rāśika*	Compound proportion	Principal, time, interest, rate	Finance, interest calculation	Interest on loans, capital investment
Rule of Seven	*Sapta Rāśika*	Multi-variable proportion	Quantity, dimensions, price	Trade, craft production	Cloth pricing by size, commodity valuation
Rule of Nine	*Nava Rāśika*	Extended compound proportion	Number, dimensions, cost factors	Manufacturing, transport, construction	Timber pricing, material scaling
Rule of Eleven	*Ekādaśa Rāśika*	Complex proportional scaling	Quantity, dimensions, distance, cost	Transport, logistics, taxation	Freight charges, caravan transport
Barter & Exchange	*Bhāṇḍa Pratibhāṇḍa*	Value equivalence	Price, quantity, commodity type	Market exchange, trade negotiation	Exchange of spices, metals, grains

6. Mathematical Thought and Socio Economic Context

The persistence of *Rāśikaṃ* methods across multiple treatises indicates their practical significance. These rules were not merely pedagogical exercises but tools for daily life computation. They reflect an intellectual culture in which mathematics served administration, commerce, and daily livelihood.

Thus, *Vyavahāra Gaṇita* demonstrates that ancient classical Indian mathematics was deeply applied in orientation, integrating theoretical clarity with economic utility.

CONCLUSION

The present study highlights that the computational procedures embedded in *Rāśikaṃ* based methods formed a coherent framework for solving financial and everyday problems in ancient classical Indian mathematics. From the Rule of Three to compound proportional rules and barter exchange, these techniques demonstrate a systematic approach to proportional reasoning rooted in practical needs such as trade, measurement, transport, and interest calculation. The works of **Āryabhaṭa** and **Brahmagupta**, along with later mathematicians, show that *Vyavahāra Gaṇita* was not merely theoretical arithmetic but an applied science closely connected with economic and social life.

The study also reveals that the expansion from simple proportion to multivariable compound rules reflects an evolving mathematical sophistication aimed at addressing increasingly complex commercial situations. These procedures anticipate modern proportional and algebraic reasoning, illustrating the continuity between classical Indian computational thought and contemporary financial mathematics.

Thus, *Rāśikaṃ* associated *sūtras* represent not isolated rules but an integrated tradition of applied mathematical reasoning that underlines India's significant contribution to the historical development of practical arithmetic.

ISSN 2394 - 7780

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