International Journal of Advance and Innovative Research
Volume 13, Issue 2: April - June 2026
ISSN 2394 - 7780

APPLICATION OF MATHEMATICAL TOOLS AND MODELS IN URBAN WASTE MANAGEMENT SYSTEMS

Dr. Sheeja K
Assistant Professor, Head of Department of Computer Science, SIES (Nerul) College Of Arts, Science and Commerce, Nerul, Navi Mumbai,
sheejak@sies.edu.in.

ABSTRACT
Urban waste management is growing challenge globally with projected waste generation reaching 3.4 billon tonnes by 2050. In India 52 tonnes of waste are generated annually with 50% being organic waste leading to significant landfill and methane emissions. Rapid urban growth has led to a significant increase in solid waste generation, creating challenges for efficient collection, transportation, and disposal. Conventional waste management practices often rely on fixed schedules and limited planning, which can result in higher operational costs and inefficient use of resources. This study explores the use of mathematical tools and models to improve the performance of urban waste management systems. The integration of these mathematical approaches provides a systematic framework for improving decision-making in waste management systems. The proposed model aims to reduce operational costs, optimize resource utilization, and enhance overall efficiency. The study demonstrates the practical value of mathematical modelling in developing sustainable and effective solutions for urban waste management challenges.

*Keywords: LFG-landfilling Gas, Methane Emission, First-Order Decay Model**,** MSW-municipal solid waste*

1. INTRODUCTION
The boom in population and urbanization have resulted in a high rate of solid waste generation in urban areas, which has made waste management a major concern to the urban authorities. Ineffective collection systems, unthoughtful transportation paths, and insufficient distribution of resources tend to increase the cost of operations and environmental issues. Conventional methods of waste management are usually based on fixed schedules and manual planning which might not be flexible to the evolving urban environment.

The mathematical tool and model have become relevant in recent years to solve these problems. Mathematical methods offer an organised and quantitative means of analysing the waste generation patterns, optimising the collection processes and enhancing the efficiency of the system in general. Using data-driven approaches, one can make a decision that is informed, which will improve performance and sustainability[8] .

This paper is about the application of different mathematical approaches to the management of waste in the city with the intention of coming up with a framework that incorporates forecasting, optimization and analysis. This practice can help in reducing the cost, enhancing the use of resources, and service delivery within the urban setting.

Methane Emission from MSW Disposal Sites.
Most of the waste disposal sites in the country are uncontrolled dumps. These sites are constant threat to ground water contamination and emits several gases including methane. Due to various variable factors, it becomes difficult to estimate correct quantities of such gaseous emissions. With this background, CPCB instituted studies on estimation of landfill gases in collaboration with IARI and NEERI. Organic matter content in the deposited MSW at the landfill site tends to decompose anaerobically leading to emission of volatile organic compounds and gaseous by products. Emission of gaseous products from landfills commonly called landfill gas (LFG) contains methane and carbon dioxide as major constituents. LFG has potential for non-conventional energy, which also contributes to greenhouse gas effect, if not managed properly. The study involved development of methodology for monitoring LFG emissions from the landfill at Nagpur and validation of methodology at other landfill sites.

2. LITERATURE REVIEW:
The rising amount of municipal solid waste (MSW) in cities has emerged as a major environmental and management issue. The high rate of population increase and a shift towards new consumption patterns has escalated the burden on the available waste management systems. In most Indian cities, the disposal of waste is still mostly based on open dumping that causes severe environmental challenges, such as groundwater pollution and greenhouse gases emissions.

Of special concern among these emissions is the methane that is produced by landfills sites as it has a high potential of global warming. Landfill gas is formed as a result of the anaerobic breakdown of organic material and is mainly made up of methane and carbon dioxide. Research conducted by the Central Pollution Control Board has shown that the estimate of the emission is complicated due to the fact that it is influenced by a number of factors including waste composition, weather conditions and landfill methods [1].

In order to counter this, mathematical models have been extensively used. The model of first-order decay is widely employed in estimating the production of methane because the former takes into account the gradual degradation of organic matter with time. Studies conducted in partnership with Indian Agricultural Research Institute and National Environmental Engineering Research Institute have helped to customize such models to the conditions in India, emphasizing the significance of location-specific parameters [3].

Besides the estimation of emissions, mathematical methods are also used to enhance the management of wastes. The trends of waste generation are analyzed and predicted through statistical methods and the design of effective collection and transportation systems is achieved through optimization. Network-based models also aid in the planning of routes based on the representation of urban infrastructure as a network [4].

A 2024 study published in Nature Sustainability highlights that global landfill methane emissions may be significantly underestimated, accounting for nearly 10% of anthropogenic methane emissions worldwide . This finding emphasizes the need for improved modeling techniques and more reliable data collection methods. Similarly, research in Waste Management (2024) comparing different emission estimation methods found that the first-order decay model aligns more closely with observed trends than alternative reporting approaches, although discrepancies still exist due to assumptions about gas collection efficiency [2].

The uncertainty in the amount of waste generated and the environmental conditions have resulted in the application of probabilistic and simulation-based methods. These techniques enable the superior assessment of various situations and enhance the effectiveness of decision making. On the whole, the available literature indicates that mathematical tools may be used to significantly contribute to the estimation of the environmental consequences and improving the performance of waste management systems [5]. Nevertheless, one still has to implement these strategies in a combination form, especially in big and complicated cities.

Overall, recent literature indicates a shift from simple estimation models toward integrated approaches that combine mathematical modelling, field measurements, and advanced technologies [6]. While the first-order decay model remains a fundamental tool, its effectiveness can be significantly improved by incorporating real-time data, probabilistic analysis, and modern monitoring techniques. This highlights the growing importance of developing comprehensive and data-driven frameworks for sustainable urban waste management [7].

3. RESEARCH METHODOLOGY:

Mathematical Approach for Methane Emission from Landfills

1. First-Order Decay Model (Core Model)

Methane generation from landfill waste is commonly modelled using a **first-order decay equation**, where organic waste decomposes over time [9].

$$Q_t = \sum_{i=1}^{n} W_i \cdot L_0 \cdot k \cdot e^{-k(t-t_i)}$$

Where:

- Q_t= Methane generation rate at time t
- W_i= Waste deposited in year i
- L_0= Methane generation potential (m³ CH_4/tonne waste)
- k= Decay rate constant (depends on climate, moisture, waste type)
- $t - t_i$= Age of waste

2. Methane Generation Potential

Methane potential depends on biodegradable organic content in MSW.

$$L_0 = MCF \times DOC \times DOC_f \times F \times \frac{16}{12}$$

Where:

- **MCF** = Methane correction factor
- **DOC** = Degradable organic carbon
- **DO**C_f = Fraction of DOC that decomposes
- **F** = Fraction of methane in landfill gas (≈ 0.5)

3. Landfill Gas Composition Model

Landfill gas typically contains:

- Methane (CH_4) ≈ 50–60%
- Carbon dioxide (CO_2) ≈ 40–50%

$$LFG = CH_4 + CO_2 + Trace\ Gases$$

4. Emission Estimation with Collection Efficiency
Not all methane escapes; some is captured.

$$E = Q_t \times (1 - \eta)$$

Where:

- E= Actual methane emitted
- η= Collection efficiency (0–1)

5. Uncertainty Modelling
Since emissions vary:

- Model parameters k, L_0as random variables
- Use probability distributions:
  - Normal distribution for waste variation
  - Uniform distribution for uncertain parameters

6. Time-Based Emission Forecasting
You can compute methane emission over years:

$$Q(t) = Q_0 e^{-kt}$$

4. Analysis:

Let us consider a Case Study:
Using a first-order decay model, methane generation from landfill waste was estimated. For a waste input of 100,000 tonnes per year, the methane generation rate after 5 years was found to be 389,400 m³/year. Considering a collection efficiency of 60%, the actual emission was reduced to 155,760 m³/year."

The numerical example below illustrates the application of the first-order decay model for a single waste input, while the Mumbai dataset extends the same model to multiple years, incorporating varying waste quantities and time-dependent decay for more realistic estimation.

a. Annual Waste Deposition Data (Mumbai)

Year	Waste Generated (TPD)	Annual Waste (tonnes/year)
2018	9,400	3,431,000
2019	9,600	3,504,000
2020	8,800	3,212,000
2021	9,200	3,358,000
2022	9,500	3,467,500
2023	9,800	3,577,000

(TPD × 365 = Annual waste)

b. Landfill & Methane Parameters (Mumbai Conditions)

Parameter	Symbol	Value	Remarks
Methane generation potential	L_0	100 m³/tonne	Typical Indian MSW
Decay rate constant	(k)	0.05 year⁻¹	Tropical climate
Methane fraction	(F)	0.5	Standard LFG
Collection efficiency	η	0.5–0.6	Mumbai average
Organic content (DOC)	Degradable organic carbon	0.12–0.18	CPCB range

We use first order decay formula to find $Q_t = \sum_{i=1}^{n} W_i \cdot L_0 \cdot k \cdot e^{-k(t-t_i)}$, using excel we could find the sample methane calculation for a dataset as shown below:

Sample Methane Calculation Dataset

Year	Waste (tonnes)	Age (years)	Qt (m³/year)
2018	3,431,000	5	=W*L0*k*EXP(-k*t)
2019	3,504,000	4	formula
2020	3,212,000	3	formula
2021	3,358,000	2	formula
2022	3,467,500	1	formula
2023	3,577,000	0	formula

Methane emission over years are found to as follows

Final Output Table

Year	Methane Generated (m³/year)	Emission (after 60% capture)
2018	~13,400,000	~5,360,000
2019	~14,000,000	~5,600,000
2020	~13,000,000	~5,200,000
2021	~14,500,000	~5,800,000
2022	~16,000,000	~6,400,000
2023	~17,800,000	~7,120,000

5. CONCLUSION:

This study demonstrates the significance of mathematical tools and models in addressing the growing challenges of urban waste management. The application of statistical, optimization, and modelling techniques provides a structured and quantitative framework for improving waste collection, transportation, and disposal processes. In particular, the use of the first-order decay model enables effective estimation of methane emissions from landfill sites, which is essential for understanding environmental impacts and planning mitigation strategies.

The analysis, supported by both a numerical example and a Mumbai-based dataset, highlights how mathematical modelling can be applied to real-world scenarios for better decision-making. The results indicate that methane emissions from municipal solid waste are substantial but can be significantly reduced through efficient gas collection systems and optimized waste management practices.

Overall, the integration of mathematical approaches enhances operational efficiency, reduces environmental risks, and supports sustainable urban development. The study emphasizes the need for adopting data-driven and model-based strategies in modern waste management systems. Future work can focus on incorporating real-time data, advanced simulation techniques, and machine learning methods to further improve prediction accuracy and system performance.

REFERENCES

[1] Central Pollution Control Board. (Year). *Municipal solid waste management annual report*. Government of India.

[2] Y. Wang, et al. (2024). Methane emissions from landfills differentially underestimated worldwide. *Nature Sustainability*.

[3] P. Kumar, et al. (2024). Detection and quantification of methane emissions from landfills. *Atmospheric Measurement Techniques*.

ISSN 2394 - 7780

[4] S. Mor, K. Kaur, & K. Ravindra. (2024). Methane emissions and waste-to-energy potential in India. *Frontiers in Sustainable Cities*.

[5] H. Scharff, et al. (2023). Impact of landfill management on methane emissions. *Waste Management & Research*.

[6] D. Grégoire, et al. (2023). Microbial methane cycling in landfills. *Nature Communications*.

[7] N. Balasus, et al. (2024). Satellite monitoring of landfill methane emissions. *arXiv preprint*.

[8] Giordano, F. R., Fox, W. P., & Horton, S. B. (2014). *A first course in mathematical modeling* (5th ed.). Brooks/Cole, Cengage Learning.

[9] Edwards, C. H., & Penney, D. E. (2008). *Differential equations and boundary value problems: Computing and modeling* (4th ed.). Pearson Prentice Hall.